that is, the indirect effect odds ratio uses the same formula as the indirect effect with a continuous outcome, but exponentiated.

When the treatment variable is continuous, the indirect effect odds ratio of (8.39) is modified as

$$TNIE(OR) = e^{(\beta_1\ \gamma_1 + \beta_3\ \gamma_1\ x_1)(x_1 - x_0)}, \tag{8.40}$$

for a change from x_0 to x_1. For example, x_0 may represent the mean of the treatment and x_1 may represent the mean plus one standard deviation, so that $x_1 - x_0$ corresponds to one standard deviation for the continuous treatment variable.

The expression for the direct effect odds ratio is, however, using a more complex formula than merely exponentiating the continuous outcome counterpart. The direct effect odds ratio is (VanderWeele & Vansteelandt, 2010; Valeri & VanderWeele, 2013)

$$PNDE(OR) = e^{(\beta_2 + \beta_3\ \gamma_0 + \beta_3\ \gamma_1\ x_0 + \beta_3\ \gamma_2\ c + \beta_3\ \beta_1\ \sigma_m^2)(x_1 - x_0) + 0.5\ \beta_3^2\ \sigma_m^2 (x_1^2 - x_0^2)}. \tag{8.41}$$

When there is no treatment-mediator interaction, $\beta_3 = 0$, the expression simplifies to

$$e^{\beta_2 (x_1 - x_0)}. \tag{8.42}$$

For a 0/1 treatment variable this results in e^{β_2}.

8.1.6 Causal effects with multiple mediators and a binary outcome

VanderWeele and Vansteelandt (2013) discuss causal effects with multiple mediators and a binary outcome. The combined effect of the mediators is considered as opposed to path-specific effects. Effects are derived using logistic regression under the rare outcome assumption. Nguyen et al. (2016) consider the combined effect of multiple mediators using probit regression without the rare outcome assumption which leads to effect formulas similar to those of Section 8.1.3 except for using a covariance matrix for the residuals of the mediators.

8.1.7 Example: Intention to use cigarettes

MacKinnon et al. (2007) analyze the model shown in Figure 8.3. This example has a binary exposure variable tx, a continuous mediator variable